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1.2 **Refractive index**

**a** Consider light of free-space wavelength 1300 nm traveling in pure silica medium. Calculate the phase velocity and group velocity of light in this medium. Is the group velocity ever greater than the phase velocity?

From *n* and *Ng* vs.*λ*curves, at*λ*= 1300 nm

*n* = 1.447 and *Ng* = 1.462

Phase velocity : *v* = *c/n* = (3×108 m s-1) / 1.447 = **2.073×108 m s-1**

Group velocity: *vg* = c/*Ng* = (3×108 m s-1) / 1.462 = **2.052×108 m s-1**

For glasses, *dn/dλ*is negative so that *Ng* > *n* and hence *vg* < *v*. Note that *vg* > *v* in a medium that will have a positive *dn/dλ.* For example, PbS, PbTe, PbSe in the region *λ*= 1 ~3.5 μm.

**b** What is the Brewster angle (the polarization angle *θp*) and the critical angle (*θc*) for total internal reflection when the light wave traveling in this silica medium is incident on a silica/air interface. What happens at the polarization angle?

The polarization (the Brewster) angle is

 * θp* = arctan(*n*2/*n*1) = arctan(1/ 1.447) = **34. 65°**

At this angle of incidence, *r*// = 0, the reflected wave has an *E*-field only perpendicular to theplane of incidence.

The critical angle for TIR is,

 *θc =* arcsin*(n*2*/n*1*) =* arcsin(1/ 1.447) = **43.72°**

**c** What is the reflection coefficient and reflectance at normal incidence when the light beam traveling in the silica medium is incident on a silica/air interface?

For light traveling in glass incident on the glass-air interface at normal incidence,

*r* ＝*r*// ＝*r*⊥$=\frac{n1-n2}{n1+n2}＝\frac{1.447-1}{1.447+1}＝0.183$

Thus, *R* = *r*2 = (0.183)2 = 0.0335

**d** What is the reflection coefficient and reflectance at normal incidence when a light beam traveling in air is incident on an air/silica interface? How do these compare with part (c) and what is your conclusion?

For light traveling in air incident on the air-glass interface at normal incidence

*r* ＝*r*// ＝*r*⊥$=\frac{n1+n2}{n1-n2}＝\frac{1.447+1}{1.447-1}＝-0.183$

*R* = *r*2 = (0.183)2 = **0.0335**

There is a 180°phase change as *r* is negative. Notice that in both cases the amount of reflection(3.35%) is the same.

1.4 **Antireflection coating**

**a** Consider three dielectric media with flat and parallel boundaries with refractive indices *n*1, *n*2 and *n*3. Show that for normal incidence the reflection coefficient between layers 1 and 2 is the same as that between layers 2 and 3 if *n*2 = [*n*1*n*3]. What is the significance of this?

For light traveling in medium 1 incident on the 1-2 interface at normal incidence,

*r*12$ =\frac{n1-n2}{n1+n2}＝\frac{n1-\sqrt{n1n3}}{n1+\sqrt{n1n3}}＝\frac{1-\sqrt{\frac{n3}{n1}}}{1+\sqrt{\frac{n3}{n1}}}$

For light traveling in medium 2 incident on the 2-3 interface at normal incidence,

*r*23$ =\frac{n2-n3}{n2+n3}＝\frac{\sqrt{n1n3}-n3}{\sqrt{n1n3}+n3}＝\frac{\sqrt{\frac{n1}{n3}}-1}{\sqrt{\frac{n1}{n3}}+1}＝\frac{1-\sqrt{\frac{n3}{n1}}}{1+\sqrt{\frac{n3}{n1}}}$

thus, *r*23 = *r*12

Significance? For an efficient antireflection effect, waves *A* (reflected at 1-2) and *B* (reflected at 2-3) in Figure 1Q4 below should interfere with near “total destruction”. That means they should have the same magnitude and that requires that the reflection coefficient between 1 and 2 should be the same as that between 2 and 3; *r*12 = *r*23. Thus, the layer 2 can act as an antireflection coating if its index *n*2 = (*n*1*n*3)1/2. This can be achieved by *r*12 = *r*23.

**b** Consider a Si photodiode that is designed for operation at 900 nm. Given a choice of two possible antireflection coatings, SiO2 with a refractive index of 1.5 and TiO2 with a refractive index of 2.3 which would you use and what would be the thickness of the antireflection coating? The refractive index of Si is 3.5. Explain your decision.

The best antireflection coating has to have a refractive index *n*2 such that *n*2 = (*n*1*n*3)1/2 =

[(1)(3.5)]1/2 = 1.87. Given a choice of two possible antireflection coatings, SiO2 with a refractive index of 1.5 and TiO2 with a refractive index of 2.3, both are close .

The phase change for wave *B* going through the coating of thickness d is 2*k*2*d* where

 *k*2 = *n*2*k*o and *k*o = wavevector in free space = 2*π*/*λ*. This should be 180°or*π*. Thus we need 2*n*2(2*π*/*λ*)*d* =*π*or

For SiO2 *d=*$\frac{λ}{4n2}=\frac{900×10^{-9}m}{4(1.5)}=0.15μm$

For TiO2 *d=*$\frac{λ}{4n2}=\frac{900×10^{-9}m}{4(2.3)}=0.10μm$

1.6 **TIR and polarization at water-air interface**

**a** Given that the refractive index of water is about 1.33, what is the polarization angle for light traveling in air and reflected from the surface of the water?

For light traveling in air towards the sea surface, *n*1 = 1 and *n*2 = 1.33

 * θp* = arctan(*n*2/*n*1) = arctan(1.33/ 1) = **53.1°.**

At this angle of incidence, *r*// = 0, the reflected wave has an *E*-field only perpendicular to theplane of incidence.

**b** Consider a diver in sea pointing a flash light towards the surface of the water. What is the critical angle for the light beam to be reflected from the water surface?

For light traveling in the water, *n*1 = 1.33 and *n*2 = 1. The critical angle for TIR for light traveling under water and hitting the surface of water is

* θc* = arcsin(*n*2/*n*1) = arcsin(1/ 1.33) = **48.75°**

1.13 **TIR and FTIR**

**a** By considering the electric field component in medium *B* in Figure 1.20 (b), explain how you can adjust the amount of transmitted light.

Consider the prism *A* when the neighboring prism *C* in Figure 1.20 (b) in far away. When the light beam in prism *A* is incident on the *A/B* interface, hypotenuse face , it suffers TIR as *θi* > *θc*. There is however an evanescent wave whose field decays exponentially with distance in medium *B*. When we bring prism *C* close to *A*, the field in *B* will reach *C* and consequently penetrates *C*. (The tangential field must be continuos from *B* to *C*). One cannot just use the field expression for the evanescent wave because this was derived for a light beam incident at an interface between two media only; no third medium. The transmitted light intensity from *A* to *C* depends on the thickness of *B*.

**b** What is the critical angle at the hypotenuse face of a beam splitter cube made of glass with *n*1 =1.6 and having a thin film of liquid with *n*2 =1.3. Can you use 45prisms with normal incidence?

For the prism *A* in Figure 1.20 (b), *n*1 = 1.6 and *n*2 = 1.3 so that the critical angle for TIR at the hypotenuse face is

* θc* = arcsin(*n*2/*n*1) = arcsin(1.3/1.6) = **54.3°**

In this case, one *cannot* use a 45 **°**prism.

**c** Explain how a light beam can propagate along a layer of material between two different media as shown in Figure 1Q13 (a). Explain what are the requirements in the indices *n*1, *n*2, *n*3. Will there be any losses at the reflections?

If the angle of incidence *θi* at the *n*1/*n*2 layer is more than the critical angle *θc*12 and if angle of incidence *θi* at the *n*1/*n*3 layer is more than the critical angle *θ13* then the light ray will travel by TIR,zigzagging between the boundaries as sketched in Figure 1Q13 (a). For example, suppose that *n*1 = 2(thin layer); *n*2 = 1 (air) and *n*3 = 1.6 (glass),

*θc*12 = arcsin(*n*2/*n*1) = arcsin(1/2) = 38.8**°**

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and *θc*13 = arcsin(*n*3/*n*1) = arcsin(1.6/2) = 53.1**°**,

so that *θi* > 53.1will satisfy TIR. There is no loss in TIR as the magnitude of the amplitude of the reflected way is the same as that of the incident wave.

NOTE: There is an additional requirement that the waves entering the thin film interfere constructively,otherwise the waves will interfere destructively to cancel each other. Thus there will be an additional requirement, called the *waveguide condition*, which is discussed in Chapter 2.

**d** Consider the prism coupler arrangement in Figure 1Q13 (b). Explain how this arrangement works for coupling an external light beam from a laser into a thin layer on the surface of a glass substrate. Light is then propagated inside the thin layer along the surface of the substrate. What is the purpose of the adjustable coupling gap?

The light ray entering the prism is deflected towards the base of the prism. There is a small gap between the prism and the thin layer. Although the light arriving at the prism base/gap interface is reflected, because of the close proximity of the thin layer, some light is coupled into the thin layer per discussion in **a** due to frustrated TIR. This arrangement is a much more efficient way to couple the light into the thin layer because the incident light is received by the large hypotenuse face compared with coupling the light directly into the thin layer.

1.16 **Diffraction by a lens**

Any lens in practice is an aperture and the image of a point is therefore a diffraction pattern. Suppose a lens with a diameter of 2 cm has a focal length of 40 cm. Suppose that it is illuminated with a plane wave, a collimated beam of light, of wavelength 590 nm. What is the diameter of the Airy disk at the focal point? What is your conclusion?

The angular position **of the first dark ring is determined by the diameter *D* of the aperture and the wavelength *λ*, and is given by

 sinθ=1.22$\frac{λ}{D}$

Since θis small, θ=sinθ=1.22$\frac{λ}{D}$=1.22$\frac{590×10^{-9}}{2×10^{-2}}$=3.6×$10^{-5}$ rad.

From the Rayleigh criterion this is also the resolving power △θmin of the lens.

If *f* = focal length of the lens, the *radius r* of the central Airy disk is determined by

 θ= *r/f*

∴ *r* = *f*θ= (40× $10^{-2}$m)( 3.6× $10^{-5}$ rad) = 1.44× $10^{-5}$m or 14.4 μm.

For nearly all practical purposes, this 29μm diameter spot at the focal plane is a *point*.